

Developing a novel SRP-FPO fuzzy model for the Structured Ranking Process of Fuzzy Multi-Criteria Portfolio Optimization Under Uncertainty

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Abstract

This paper aims to develop a systematic and transparent decision-making model for optimizing stock portfolios under uncertainty using fuzzy logic. The research intends to evaluate and choose Exchange-Traded Funds (ETFs) by integrating performance indicators and risk measurements through the SRP-FPO (Structured Ranking Process for Fuzzy Portfolio Optimization) model. The model introduces fuzzy weights for five criteria—CAGR, Volatility, Sharpe Ratio, Max Drawdown, and Liquidity—utilizing triangular fuzzy numbers. Normalized scores are compiled using SRP methodology to produce a ranked list of ETFs. The SRP-FPO framework effectively ranks ETFs by addressing ambiguous preferences and trade-offs, indicating alignment with actual market dynamics and investor anticipations. Using the SRP method, we created a ranked list of ETFs, with their positions determined by their combined scores. The new model improves the Structured Ranking Process (SRP) by integrating fuzzy logic into portfolio decision-making, offering a novel method for financial analytics. The model is applicable in Robo-advisory systems, sustainable investing, and individual portfolio design under risk uncertainty. This is the first formal fuzzy extension of SRP applied to financial portfolio

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optimization, known as Structured Ranking Process for Fuzzy Portfolio Optimization (SRP-FPO).

Keywords: ETF Evaluation, Fuzzy MCDM, Investment Decision-Making, Portfolio Optimization, Structured Ranking Process

1. Introduction

Portfolio optimization has improved due to advancements in both the theoretical and practical domains. Markowitz's work on mean-variance analysis laid the groundwork for later research. Konno and Yamazaki (1991) then expanded on this by using models based on mean-absolute deviation. Jorion (1992) used the standard methods of the time, but Michaud & Ma (2001) improved asset allocation techniques. While Davendralingam and DeLaurentis (2015) identified the relevance of robust portfolio selection for optimization-based system-of-systems architectures. While Guerard et al. (2024) update Markowitz's mean-variance efficiency for the US and Japan markets. Particle swarm optimization (PSO) is analyzed by Mba & Mai (2022). Konstantinov and Fabozzi develop a structural network theory for portfolio management. Thakkar and Chaudhari (2021) and Kolm et al. (2014) review PSO applications and current portfolio optimization trends. Meanwhile, Michaud and Ma (2001) provide asset allocation advice, and Li and Hoi (2014) study dynamic online portfolio selection. Pai & Michel (2009) optimize small portfolios with evolutionary k-means clustering, Bradshaw et al. (2009) constrained PSO methodologies. Armanianzas and Lozano (2005) were the first to develop multi-objective evolutionary techniques, whereas Konno and Yamazaki (1991) introduced the MAD model. The studies demonstrate the evolution of traditional portfolio optimization methods into computational intelligence approaches.

The emergence of evolutionary algorithms is attributed to Ponsich et al. (2012), who explored multi-objective methodologies, and Armanianzas & Lozano (2005), who pioneered multi-objective evolutionary methods. In addition, Bradshaw et al. (2009) and Pai & Michel (2009) enhanced evolutionary methods for portfolio diversification. Bio-inspired algorithms achieved significance due to Mba and Mai's (2022) integration of PSO to cryptocurrency. Davendralingam and DeLaurentis (2015) built robust

optimization methodologies for complex systems, while Fabozzi et al. (2007) examined portfolio management. Machine learning transformed the field of research, with Hieu (2020) and Jang & Seong (2023) utilizing deep reinforcement learning, while Ta et al. (2020) combined LSTM networks with Optimization. Contemporary surveys by Thakkar & Chaudhari (2021) and Priyadarshi & Kumar (2025) document these advancements, while Kolm et al. (2014) provide a historical perspective. This progression demonstrates the field's continuous integration of traditional finance theory with cutting-edge computational techniques. Zanjirdar's 2020 study provides a thorough analysis of optimization models. Contemporary methodologies are built on foundational works. Gunjan & Bhattacharyya (2023) recently conducted an exhaustive review of modern techniques, while Thakkar & Chaudhari (2021) documented progress in particle swarm optimization applications. With a Bayesian approach to global optimization, Black & Litterman (1992) revolutionized the fundamental principles of portfolio optimization, which continue to influence the field today. This discipline evolved through extensive examination by Best (2010) and the comprehensive frameworks offered by Fabozzi et al. 2007. Jin's (2025) use of Markowitz models, along with Chaweewanchon and Chaysiri's (2022) work on using machine learning for stock selection are examples of current research in mean-variance analysis. The development of computational methods has significantly impacted the field. Ma et al. (2020)'s deep learning techniques demonstrate how early neural network applications by Freitas et al. (2009) have advanced into sophisticated AI-driven models.

Yu et al. (2019) concept of Reinforcement learning is often highlighted as a powerful tool, particularly in the context of dynamic optimization, as shown by recent research. Ban et al. (2018) demonstrate the transformative potential of machine learning. Examining the intricate financial market uncovers advanced portfolio optimization based on theoretical principles. Furthermore, it suggests the potential integration of traditional financial theories with cutting-edge machine learning and optimization algorithms to develop sophisticated tools for risk assessment and return improvement across diverse market conditions. The findings suggest a significant evolution in the practices of management of assets.

The switch from mean-variance to machine learning has highlighted fundamental flaws in portfolio optimization theory and practice. Despite their theoretical soundness,

Markowitz's efficient frontier and the Black-Litterman framework sometimes fail to account for market reality. Challenges include regime changes, liquidity constraints, and abnormal return distributions. Second, evolutionary algorithms and bio-inspired optimization approaches can solve multi-objective problems, but they are computationally intensive and lack consistent evaluation criteria. As machine learning, particularly deep reinforcement learning and neural networks, gain popularity, model interpretability, over fitting, and robustness in volatile markets become commonplace. The purpose of this research is to establish a structured fuzzy decision framework for investment selection under uncertainty to overcome the limitations of standard portfolio optimization methods. The objectives mainly include: (1) to develop a Structured Ranking Process-based Fuzzy Portfolio Optimization (SRP-FPO) model that integrates fuzzy logic and multi-criteria decision-making to assess and rank investment alternatives during uncertain financial conditions; (2) to serve as a systematic multi-criteria evaluation framework for portfolio selection using key economic variables such as return, risk, liquidity, and volatility in quantitative and qualitative terms; (3) to empirically evaluate the proposed SRP-FPO model on real financial market data from Yahoo Finance and establish its value by generating transparent and interpretable portfolio rankings; and (4) to compare the proposed structured fuzzy ranking strategy with existing portfolio analysis methods with respect to decision transparency, computational simplicity, and ability to handle uncertainty in financial data.

In order to fulfil these objectives, we propose a five-stage Simple Ranking Process (SRP) intended to evaluate exchange-traded funds (ETFs) based on key financial metrics that include compound annual growth rate (CAGR), volatility, Sharpe ratio, maximum drawdown, and liquidity. In the first stage, historical data for five major ETFs—SPY, QQQ, TLT, IWM, and GLD is sourced from Yahoo Finance. Min-max normalization of data is followed by inverse weight ranking, highlighting CAGR and volatility as primary factors. Later, normalized values are integrated with criterion weights for the computation of weighted scores. IALO, heatmaps, Sankey diagrams, and parallel coordinate plots are used for the visual representation of the findings in the last phase, effectively illustrating performance trade-offs. This Python-based methodology serves as an effective tool in an investor's toolbox for an investing strategy that ascertains reproducibility, besides

providing a transparent, data-driven framework for well-informed portfolio optimization decisions.

This study demonstrates significant improvements in portfolio optimization and ETF selection. The framework offers a systematic Simple Ranking Process (SRP) that incorporates a range of financial measures, including CAGR, Sharpe Ratio, and volatility, into a transparent, criteria-weighted decision model, presenting a viable alternative to classic mean-variance optimization. This study verifies the SRP approach by examining five prominent ETFs (SPY, QQQ, TLT, IWM, and GLD). It demonstrates that SPY exhibits superior risk-adjusted returns (Sharpe Ratio: 0.94) and liquidity, while also emphasizing TLT's inefficiencies, as evidenced by its negative CAGR. The present investigation contributes to the field of visual analytics within finance through the introduction of innovative heatmaps, Sankey diagrams, and parallel coordinates plots. These visualizations facilitate investors' assessment of the interrelationships between return, risk, and liquidity. Furthermore, this research provides actionable insights for portfolio managers by employing composite scores to rank exchange-traded funds (ETFs), a methodology enabled by a reproducible Python-based workflow. These contributions link quantitative finance with decision science, providing theoretical rigor and practical applicability for data-driven investing strategies.

2. Theoretical Underpinning

This study uses Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) to assess alternatives among uncertainty, referencing decision-making research (Nădăban et al., 2016; Behzadian, 2012). According to Sun (2010) and Chu (2002), fuzzy set theory regulates linguistic and ambiguous data. It begins using evaluation criteria and options from domain-specific applications, including supplier selection (Junior et al., 2014) and risk assessment (Wang & Elhag, 2006). Second, we use fuzzy AHP (Kutlu & Ekmekçioğlu, 2012) to derive criterion weights, addressing subjectivity through pair-wise comparisons. Third, we create a fuzzy decision matrix and calculate the alternative's distance from fuzzy positive and negative ideal solutions (Zeng et al., 2013). We utilize validation techniques (Palczewski & Sałabun, 2019) and recent extensions (Pandey et al.,

2023) to rank alternatives based on closeness coefficients. We program the process for reproducibility and to conform to industrial applications (Yadav et al., 2018) and symmetry principles (Chou et al., 2019).

Fuzzy Analytic Hierarchy Process (AHP) improves conventional AHP by integrating fuzzy set theory to address uncertainty and ambiguity in decision-making, especially regarding subjective assessments (Chang, 1996). The method employs fuzzy numbers, typically triangular or trapezoidal, to represent linguistic preferences, allowing for more realistic pair-wise comparisons (Wang et al., 2008). The Extent Analysis Method for weight derivation (Zhu et al., 1999) and the consistency verification method (Kubler et al., 2016) represent major developments in the field. The implementation of Fuzzy AHP has been thoroughly documented across multiple domains, including energy efficiency optimization (Beşikçi et al., 2016), risk assessment (Yazdi et al., 2020), supplier selection (Ayhan, 2013), and healthcare quality evaluation (Singh & Prasher, 2019). Recent extensions, such as spherical fuzzy AHP (Kutlu Gündoğdu & Kahraman, 2020), improve the ability to model complex uncertainty. This improvement is particularly beneficial for multi-criteria group decision problems that involve imprecise human judgment.

Opricovic et al. (2004, 2011) proposed the VIKOR method, which stands for “Vise Kriterijumska Optimizacija I Kompromisno Resenje”, as an approach for MCDM that can be used for ranking and determining compromises. Mardani et al. (2016) found that it signifies competing criteria that offer solutions that maximize the group's benefit while minimizing personal discontent. This method provides a compromise solution index using normalized Manhattan (L_1) and Chebyshev (L_∞) distances, thus allowing trade-offs among different criteria (Jahan et al., 2011, 2013). Spherical fuzzy VIKOR (Kutlu Gündoğdu & Kahraman, 2019) and q-rung Ortho pair fuzzy VIKOR (Gao et al., 2020) are instances of fuzzy extensions that enhance the method's capability to manage uncertainty. Some of the applications include material selection (Girubha & Vinodh, 2012), healthcare management (Zeng et al., 2013), supplier selection (Alimardani et al., 2013), and sustainability assessments (Pourebrahim et al., 2014). VIKOR's adaptation in complicated decision-making settings can be seen by how it interacts with SWARA (Alimardani et al., 2013), TOPSIS (Shekhovtsov & Sařabun, 2020), and WASPAS (Vaid et al., 2022) in recent hybrid models. The method's rampant use in engineering,

environmental management, and company analytics is assured by its robustness in group utility maximization and conflict resolution.

3. Data Sources and Methodology

3.1. Data Sources with description

The financial data used in this study were collected from Yahoo Finance (<https://finance.yahoo.com/>), a reputable and publicly accessible financial data platform widely used in academic and industry research. The selection of the Simple Ranking Process (SRP) with fuzzy integration is appropriately justified, underlined by its ability to address the shortcomings of Blackbox multi-criteria decision-making (MCDM) methods, while also ensuring clarity and computational effectiveness's rank-based aggregation (Edwards, 1977) allows proper decision paths, in contrast to VIKOR and TOPSIS, which require advanced normalizing procedures for financial applications necessitating traceability. The fuzzy extension (Zimmermann, 2001) addresses inherent data uncertainties in ETF evaluation while dealing with mixed data types, such as crisp volatility metrics and linguistic liquidity assessments. This hybrid approach integrates methodological rigor with practical accessibility, reducing the high computational demands of AHP/ANP while dealing with rank reversal issues common in traditional TOPSIS applications. Empirical studies demonstrate their effectiveness in scenarios requiring: (1) intuitive trade-offs between conflicting financial criteria; (2) handling of heterogeneous data granularity; and (3) the inclusion of expert judgment under uncertainty, making it particularly suited for multi-criteria portfolio optimization problems that require the systematic integration of both quantitative metrics and qualitative assessments. The model's five-stage pipeline offers reproducible results while maintaining interpretability, thus avoiding the complexities associated with black-box systems, which are consistent with established practices in financial decision support systems (see Table 1).

Table 1: List of ETFs under Comparison

S/L	Description	Purpose
1	SPY (S&P 500 ETF)	Represents the broad U.S. equity market; used as a benchmark for portfolio performance and stability.
2	QQQ (NASDAQ-100 ETF)	Focuses on tech-heavy growth stocks; captures high-risk, high-reward innovation-led exposure.
3	TLT (20+ Year Treasury Bond ETF)	Provides exposure to long-term U.S. Treasury bonds; used for hedging and portfolio risk reduction.
4	GLD (Gold ETF)	Tracking gold prices serves as a safe-haven asset during market turbulence and inflation.
5	IWM (Russell 2000 ETF)	Represents U.S. small-cap stocks; offers diversification and higher growth potential at elevated risk.

Table 2: Evaluation Criteria Used in SRP Fuzzy Model

S/L	Criterion	Purpose
1	CAGR (Compound Annual Growth Rate)	Measures the annualized return of an ETF, indicating long-term growth potential.
2	Volatility	Captures the price fluctuation or risk; essential for assessing stability.
3	Sharpe Ratio	Assesses risk-adjusted return, combining return and volatility into a single metric.
4	Max Drawdown	Indicates the worst historical loss from peak to trough; critical for downside risk evaluation.
5	Liquidity	Reflects ease of trading through metrics like average volume and bid-ask spread. Higher liquidity implies lower transaction costs.

Table 3: Descriptive Statistics

	GLD	IWM	QQQ	SPY	TLT
count	250	250	250	250	250
mean	267.0643	214.6543	500.4814	577.5121	88.89237
std	29.19783	12.94129	32.13078	28.10956	3.416612
min	220.55	174.338	415.5933	495.0166	83.34763
25%	243.205	206.1441	478.1093	557.7018	86.13151
50%	256.795	216.216	503.0109	580.7546	88.06172
75%	300.78	223.3	523.1539	597.6407	90.83516
max	316.29	240.4764	568.14	637.1	97.81111

Source: Author analysis

3.2. Methodology

The proposed study comprises different stages and includes:

Stage 1: Five investment options (GLD, IWM, QQQ, SPY, and TLT) and five evaluation criteria (CAGR, Volatility, Sharpe Ratio, Max Drawdown, Liquidity) were identified, and a decision matrix (Table 2) was constructed using quantitative data. Data collected from

Yfinance (open source) include SPY (S&P 500), QQQ (NASDAQ-100), TLT (20+ Year Treasury Bonds), GLD (Gold), and IWM (Russell 2000) — representing large-cap, tech, bonds, commodities, and small-cap asset classes, respectively (refer to Table 3).

Stage 2: The criteria values were normalized and/or ranked to ensure comparability across different units and scales. This ensures that values are unit-free and comparable. Common normalization techniques include:

- For beneficial criteria (higher is better), we can compute normalized criteria values as:

$$x_{ij}^{\text{norm}} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)} \quad (1)$$

- For non-beneficial criteria (lower is better), we can compute normalized criteria values as:

$$x_{ij}^{\text{norm}} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)} \quad (2)$$

where, x_{ij} Is the value for ETF i and criterion j

Stage 3: Criterion weights were applied based on the ranks assigned in Table 1. More critical criteria, such as CAGR and Volatility, were given higher importance. In this stage, we convert ranks into weights (e.g., Rank 1 = most important), and the weighting Formula (Inverse Rank Weighting) can be computed as:

$$w_j = \frac{\frac{1}{\text{Rank}_j}}{\sum_{k=1}^n \frac{1}{\text{Rank}_k}} \quad (3)$$

where, w_j is the weight of the criterion j . Rank_j is the rank of the criterion j from Table 1

Stage 4: Weighted scores were computed for each ETF by multiplying the normalized values by the respective criterion weights and summing across all criteria.

Weighted Score per ETF:

$$S_i = \sum_{j=1}^n w_j \cdot x_{ij}^{\text{norm}} \quad (4)$$

where, S_i is the overall score for ETF i . w_j is the weight for the criterion j . x_{ij}^{norm} is the normalized value of the ETF i for criterion j .

Stage 5: The aggregated scores were used to generate the final SRP ranking. This allowed for a transparent and structured comparison, identifying the most efficient ETF aligned with investor objectives.

Table 4: Beneficial and non-beneficial Criterion

Criterion	Type	Normalization Used
CAGR	Beneficial	(1)
Sharpe Ratio	Beneficial	(1)
Liquidity	Beneficial	(1)
Volatility	Non-beneficial	(2)
Max Drawdown	Non-beneficial	(2)

Source: Author analysis

4. Analysis and Discussion

4.1. SRP Pipeline Architecture Diagram

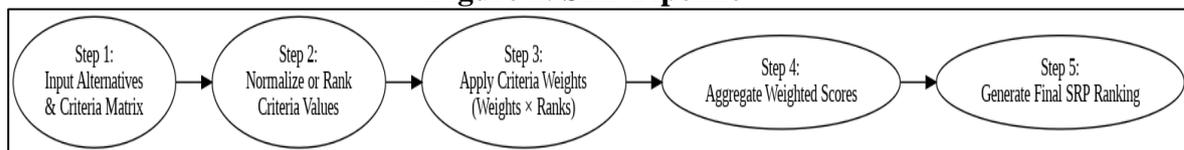
The pipeline for implementing SRP has been demonstrated to offer a comprehensive overview of the entire process—the Mermaid diagram. The SRP methodology consists of five sequential steps: defining the decision matrix, normalizing or ranking criterion values, applying weights, aggregating weighted scores, and generating final rankings. This method facilitates a clear, criteria-weighted assessment of options; promoting uniform and reliable decision-making in multi-criteria financial portfolio analysis (see Figure 1).

Step 1: Define Alternatives and Criteria

Table 1 represents a listing of alternatives (tickers) and the criteria (metrics) used in the analysis. It includes SPY (S&P 500), QQQ (NASDAQ-100), TLT (20+ Year Treasury Bonds), GLD (Gold), and IWM (Russell 2000) — representing large-cap, tech, bonds,

commodities, and small-cap asset classes, respectively. It establishes a unique correlation among five ETFs—GLD, IWM, QQQ, SPY, and TLT—and key investment performance metrics: CAGR, Volatility, Sharpe Ratio, Max Drawdown, and Liquidity. This architecture provides a practical basis for advanced decision-making methodologies, especially the Simple Ranking Process (SRP). The approach leads to consistent scoring, normalization, and rank aggregation by continuously associating each alternative with pertinent criteria. This method indicates that the SRP model integrates both expected and associated risks, offering investors the opportunity to make more informed and transparent portfolio decisions in the face of uncertainty (see Table 5).

Figure 1: SRP Pipeline



Source: Author analysis

Table 5: Alternatives and Criteria

Alternative	Criterion
GLD	CAGR
IWM	Volatility
QQQ	Sharpe
SPY	MaxDrawdown
TLT	Liquidity

Source: Author analysis

Table 6: Decision Matrix

Ticker	CAGR	Volatility	Sharpe	MaxDrawdown	Liquidity
GLD	0.112305	0.154036	0.771489	-0.22002	8336369
IWM	0.100172	0.231303	0.530251	-0.31912	30274302
QQQ	0.174705	0.233564	0.809474	-0.35119	48826555
SPY	0.160846	0.175168	0.943159	-0.24496	75397317
TLT	-0.10178	0.162078	-0.5843	-0.48351	26635893

Source: Author analysis

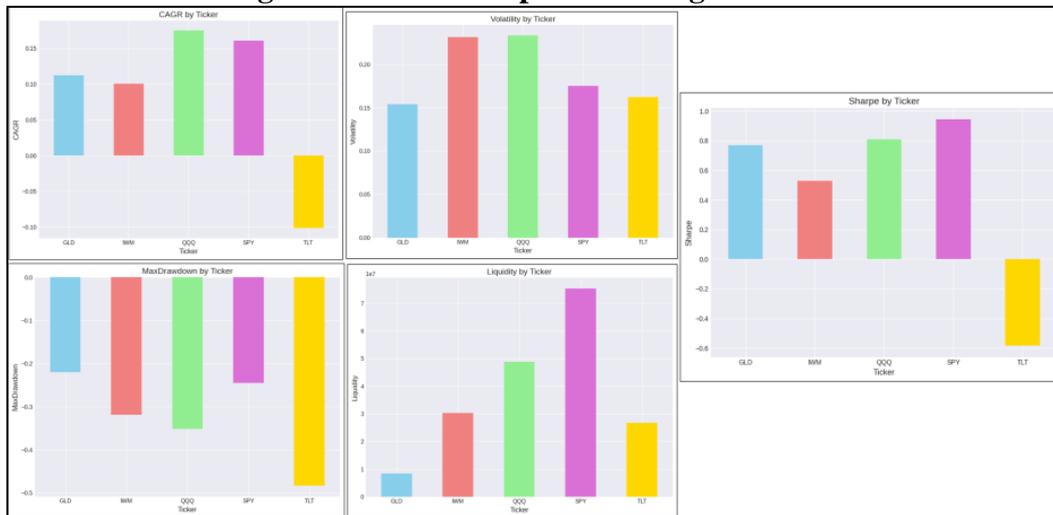
Next, we create a decision matrix based on actual metric values that correspond to each ticker and criterion. Table 2 presents the Decision Matrix for five ETFs evaluated against five critical financial metrics. SPY has been determined as the most balanced performer,

exhibiting a high CAGR of 0.1608, low volatility of 0.1751, and the highest Sharpe ratio of 0.9432, complemented by exceptional liquidity of 75M. QQQ illustrates significant growth (0.1747) and strong risk-adjusted performance (Sharpe: 0.8094), though it demonstrates the highest volatility (0.2335) and maximum drawdown (−0.3512), indicating increased downside risk. GLD offers a balanced risk-return profile, serving as a stabilizing asset. IWM provides moderate returns; however, it encounters elevated drawdown and volatility. TLT exhibits substantial underperformance, characterized by a negative compound annual growth rate (CAGR) and Sharpe ratio, demonstrating an unfavourable risk-return trade-off. These indicators together capture the various dimensions of portfolio selection and serve as the input decision matrix for the Structured Ranking Process–Fuzzy Portfolio Optimization (SRP-FPO) model. The matrix enables systematic ranking of investment alternatives under uncertainty by integrating quantitative financial indicators into a fuzzy multi-criteria evaluation framework (see Table 6).

Ticker bar graphs are created for each metric, allowing for a visual comparison of the tickers based on distinct criteria. The comparative performance charts of five ETFs demonstrate notable differences in return, risk, and liquidity characteristics. SPY excels in Sharpe ratio and CAGR, demonstrating exceptional risk-adjusted performance and steady growth. QQQ reflects this pattern, albeit with little increased volatility. GLD demonstrates modest returns accompanied by reduced volatility, serving as a viable hedge. IWM exhibits heightened volatility and drawdowns, rendering it riskier despite commendable returns. TLT exhibits subpar performance across all parameters, characterized by a negative compound annual growth rate (CAGR) and Sharpe ratio, signifying unfavourable return dynamics in the context of increasing interest rates. Liquidity is maximized for SPY and QQQ, hence improving tradability. These insights provide educated portfolio diversification by return-risk-liquidity trade-offs. The graphical research illustrates the comparative performance of selected ETFs using key financial indicators (CAGR, volatility, Sharpe ratio, maximum drawdown, and liquidity) obtained from market data via Yahoo Finance. These visualizations support the research objective of establishing a systematic multi-criteria evaluation framework by showing differences in return, risk, and market accessibility among investment options. Therefore, the results provide empirical support for the Structured Ranking Process–Fuzzy Portfolio

Optimization (SRP-FPO) framework, enabling a transparent ranking of portfolios under financial uncertainty (refer to Figure 2).

Figure 2: Ticker comparison using Bar Plots



Source: Author analysis

Step 2: Rank Each Criterion

We generate a table for ranks for each criterion by weight. The criteria ranking demonstrates the impact of performance and risk preferences on the multi-criteria assessment of exchange-traded funds (ETFs). The opportunities for investment growth for an extended duration is highlighted by a high compound annual growth rate (CAGR). Risk-adjusted returns are often measured using volatility and the Sharpe Ratio. The fourth-ranked Max Drawdown signifies capital preservation during a decline. Lower liquidity ratings indicate that strategic allocation decisions prioritize performance and stability over tradability. This prioritization framework thus supports weighted decision models; ensuring that investors focus on long-term, risk-averse returns rather than swift market access (refer to Table 7).

In addition, we generate a weighted ranking matrix heat map. This heat map highlights the weighted ranks, aimed at a clear understanding of the influence of criterion weights on the performance of each ticker across the criteria. The Multi-Criteria Influence Matrix demonstrates the relative importance of decision-making factors among five ETFs—GLD, IWM, QQQ, SPY, and TLT—according to their impact on the final ranking. GLD

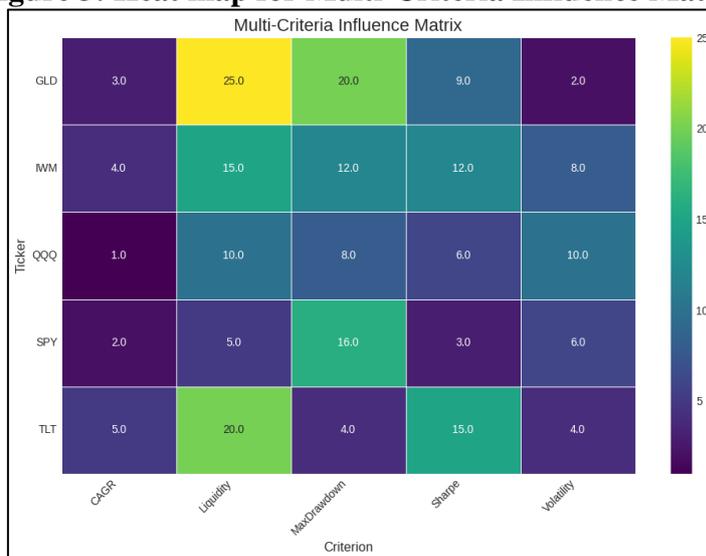
and TLT exhibit significant sensitivity to liquidity and drawdown, consistent with their defensive functions in portfolio construction. SPY highlights a balanced distribution of influence, featuring significant strength in drawdown and moderate levels of liquidity, therefore reinforcing its dominance in risk-adjusted performance. Despite giving higher returns, QQQ leads to reduced influence due to its volatility and drawdown exposure, while IWM shows a moderate profile across all criteria. The matrix exhibits the growing importance of multi-criteria frameworks for asset evaluation beyond uni-dimensional analysis. Using a structured decision environment, we learn how these key indicators impact each other while considering investing options. This matrix serves as the conceptual framework for the SRP-FPO fuzzy model; enabling systematic multi-criteria ranking of investment alternatives under financial uncertainty (see Figure 3).

Table 7: Criterion Rank

Criterion	Rank
CAGR	1
Volatility	2
Sharpe	3
MaxDrawdown	4
Liquidity	5

Source: Author analysis

Figure 3: Heat map for Multi-Criteria Influence Matrix



Source: Author analysis

Step 3: Apply criteria weights

The weighted rankings for each ticker and criterion for all ETFs are presented in the table. TLT attracted inflows due to its low volatility and modest drawdown, appealing to risk-averse investors, while SPP and QQQ exhibited robust returns, indicating their competitiveness in the market. Conversely, GLD remains stable with moderate return potential, whereas IWM faces difficulties due to its higher risk and lower Sharpe ratio. This visualization method enhances quantitative ranking and facilitates trade-off analysis, making it an essential tool for informed portfolio decisions in a complex investment environment (refer to Table 8).

Table 8: Weighted ranks for criteria

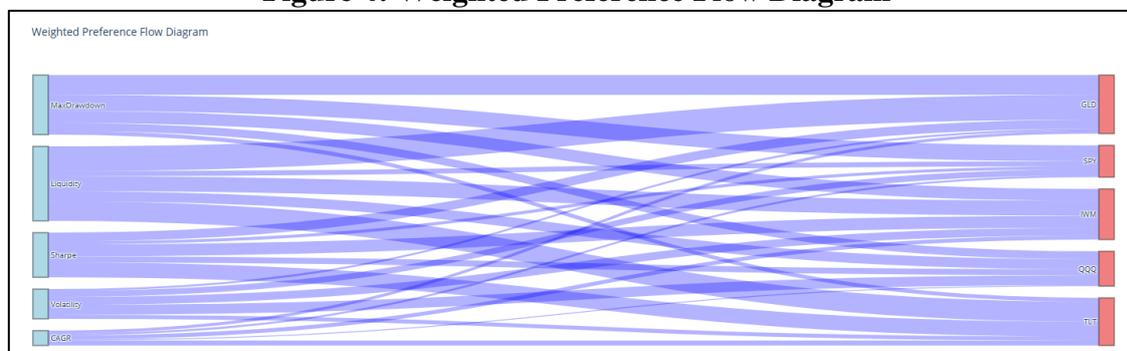
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TLT	-0.10178	0.162078	-0.5843	-0.48351142	26635893

Source: Author analysis

Next, we generate a Sankey diagram of the criteria's influence. It illustrates the impact of each criterion's weighted ranks on ticker evaluation. The Weighted Preference Flow Diagram illustrates the impact of Max Drawdown, Liquidity, Sharpe Ratio, Volatility, and CAGR on ETF rankings within a visually enhanced multi-criteria decision-making framework. The width of the flow lines indicates the choice or weight of each criterion-ETF linkage. SPY and QQQ have useful inflows across multiple performance parameters, illustrating return efficiency and risk minimization. Despite lower returns, Volatility and Max Drawdown enhance TLT's low-risk profile. Fragmented support for GLD and IWM implies niche strengths but poor balance. This Sankey-style graphic supports the parallel coordinates plot and improves the SRP framework's ability to visualize cost-benefit trade-offs for ETF selection under uncertainty. From the view of structured decision-making framework, the relative influence and weight distribution of each criterion can be understood. This also supports the research goal of implementing the SRP-FPO fuzzy

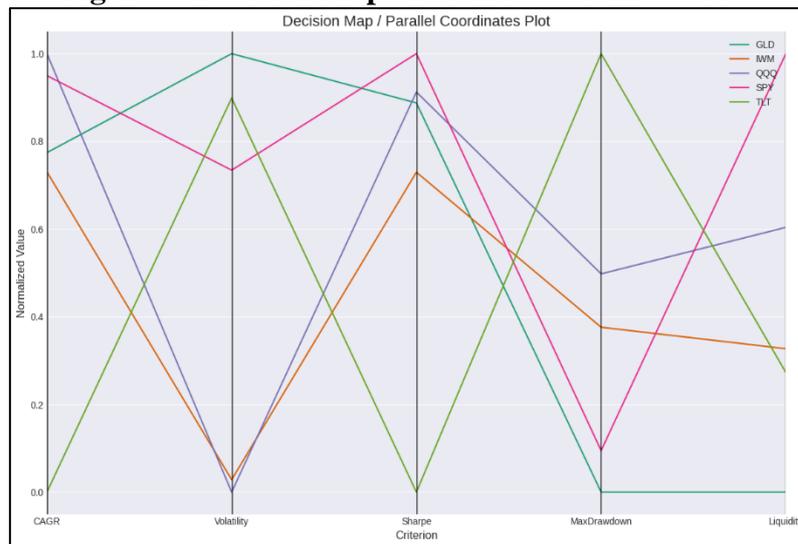
model to systematically integrate multi-criteria financial indicators for transparent portfolio ranking under uncertainty (see Figure 4).

Figure 4: Weighted Preference Flow Diagram



Source: Author analysis

We also create a Decision map / parallel coordinates plot. The decision map visualization is useful for showing the trade-offs between criteria for each ticker and how they perform across all criteria simultaneously on a normalized scale. The parallel coordinates plot presents a nuanced comparative analysis of five key ETFs—SPY, QQQ, TLT, IWM, and GLD—based on normalized values across critical investment metrics: CAGR, volatility, Sharpe ratio, maximum drawdown, and liquidity. SPY emerges as the most balanced and efficient asset, blending substantial return potential with robust risk-adjusted performance and liquidity, while QQQ also reflects this efficiency, but with increased drawdowns. TLT is effective in reducing volatility and drawdown; although it falls short in return creation, leaving it a prudent hedge. GLD and IWM exhibit disjointed profiles characterized by trade-offs between return and risk. The visualization underlines the SRP model's ability to include multi-criteria evaluation, revealing hidden patterns in cost-benefit trade-offs among various asset classes. The figure underscores the comparative trade-offs between investment options, providing for the simultaneous assessment of market accessibility, risk, and return within a multi-criteria framework. This visualization supports the research goal of using the SRP-FPO fuzzy model to methodically evaluate portfolio opportunities under uncertain financial conditions (refer to Figure 5).

Figure 5: Decision Map / Parallel Coordinates Plot

Source: Author analysis

Step 4: Aggregation of weighted scores

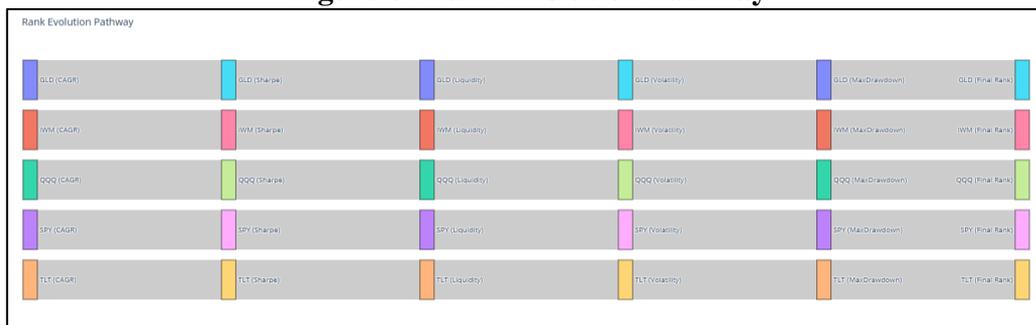
The next part of the analysis includes a table showcasing the final ranking of the tickers determined by the cost-benefit analysis, primarily the aggregate of weighted ranks. The Simple Ranking Process (SRP) model has been used for assessing five financial instruments—SPY (S&P 500 ETF), QQQ (NASDAQ-100 ETF), TLT (long-duration Treasury Bonds), IWM (Russell 2000 ETF), and GLD (Gold ETF)—by examining cost-benefit efficiency utilizing weighted rank sums. SPY gained the highest ranking with a score of 32, indicating outstanding performance in terms of return, risk, and cost metrics. Also, QQQ, with a second-place ranking, exhibited significant growth potential. TLT and IWM demonstrated moderate efficiency, likely influenced by their distinct risk-return profiles. GLD is placed at the lowest rank, suggesting minimal benefits about cost based on the selected criteria. The SRP results demonstrate the predominance of large-cap equity ETFs, specifically SPY and QQQ, for achieving optimal returns adjusted for risk in a diversified investment framework. The weighted rank sums integrate multiple evaluation criteria—CAGR, volatility, Sharpe ratio, maximum drawdown, and liquidity—to construct a systematic order among the investment alternatives. The results demonstrate the effectiveness of the proposed structure in providing an understandable and systematic multi-criteria portfolio ranking under financial uncertainty, which fulfills the primary research objectives (see Table 9).

Table 9: Final rank of tickers

Ticker	Weighted Rank Sum	Final Rank
SPY	32	1
QQQ	35	2
TLT	48	3
IWM	51	4
GLD	59	5

Source: Author analysis

Figure 6: Rank Evolution Pathway



Source: Author analysis

Step 5: Rank flow diagram

The Sankey diagram reflects the path of ranking of each ticker through the different criteria while reaching the goal of the final rank. It visualizes the rank evolution pathway for five ETFs across six evaluation criteria, culminating in a final SRP ranking. The multi-dimensional performance of each ETF—GLD, IWM, QQQ, SPY, and TLT—is demonstrated by a systematic transformation through ranks in CAGR, Sharpe, liquidity, volatility, and max drawdown. Observations indicate that GLD retains strong rankings across all metrics, which contributes to its favourable final rank. In contrast, TLT and SPY constantly lag, signalling their role in suboptimal trade-offs in terms of risk-adjusted returns. The illustration emphasizes the process by which certain ETFs achieve final dominance by optimizing returns and stability, allowing investors with an evidence-based pathway to optimize portfolio selection using SRP methodology. It highlights the incremental development of the criteria within the Structured Ranking Process, emphasizing the role of unique outcome indicators to the comprehensive portfolio ranking. This supports the research objective of using the SRP-FPO fuzzy framework to

provide an understandable and systematic ranking of investment alternatives under financial uncertainty (refer to Figure 6).

4.2. Risk-adjusted returns

Another important determinant, the Sharpe ratio, ranks SPY as the best risk-adjusted return, with QQQ and GLD following closely. TLT ranks bottom with a negative Sharpe ratio, indicating poor risk-reward performance. It also enables portfolio managers to select capital-efficient assets such as SPY and QQQ while avoiding low-yield, high-risk instruments such as TLT to obtain the greatest performance. These metrics collectively capture the various aspects of portfolio evaluation by integrating return potential, risk exposure, downside stability, and market tradability. The table provides an empirical basis for the SRP-FPO fuzzy multi-criteria framework, supporting the research's for systematically ranking investment alternatives under financial uncertainty (see Table 10).

Table 10: Risk-adjusted returns

Ticker	CAGR	Volatility	Sharpe	MaxDrawdown	Liquidity
SPY	0.160464	0.175139	0.942077	-0.244964	7.54E+07
QQQ	0.175586	0.23353	0.813342	-0.351187	4.88E+07
GLD	0.106739	0.154086	0.739091	-0.220022	8.32E+06
IWM	0.098396	0.231151	0.523748	-0.319116	3.03E+07
TLT	-0.101583	0.162136	-0.583224	-0.483511	2.67E+07

Source: Author analysis

5. Conclusion and Implications

This study employs the Simple Ranking Process (SRP) to analyze five popular ETFs—GLD, IWM, QQQ, SPY, and TLT—based on five investment criteria: CAGR, Volatility, Sharpe Ratio, Maximum Drawdown, and Liquidity. Figure 1 shows the five-step analysis: creating a criteria matrix, normalizing values, applying weights based on prioritized relevance (Table 1), and aggregating decision matrix scores (Table 2). SPY performed best with strong returns, minimal volatility, and high liquidity. However, volatility hurt QQQ's performance. TLT remained the weakest given its negative CAGR and significant drawdowns. These insights help business and portfolio managers enhance return while

limiting risk, strengthening data-driven ETF selection strategies linked with financial goals and market conditions.

Some of the major implications of the proposed model can be:

- **Strategic Portfolio Allocation:** The better risk-adjusted returns of SPY and QQQ provide investors and asset managers with the opportunity for the maximization of long-term goals with minimized drawdowns.
- **Product Positioning and Advisory:** Using Sharpe-based rankings, financial advisors can customize investment recommendations, warning against underperformers like TLT and highlighting SPY and QQQ for moderate-to-aggressive risk profiles.
- **Liquidity-Driven Trading Strategy:** High liquidity trends of SPY and QQQ may encourage bulk trading for Institutional investors.
- **Risk Mitigation Planning:** ETFs like TLT with negative Sharpe ratios and big drawdowns reflect signs for diversification into safer assets or risk buffers.
- **Data-Driven Investment Products:** Multi-criteria performance analysis frameworks (CAGR, volatility, Sharpe) can be integrated development of fund houses, to create smart-beta or thematic ETFs.

Data Collection Statement

The financial data used in this study were collected from Yahoo Finance (<https://finance.yahoo.com/>), a reputable and publicly accessible financial data platform widely used in academic and industry research. The selected assets include SPY (S&P 500 ETF), QQQ (NASDAQ-100 ETF), TLT (20+ Year Treasury Bond ETF), GLD (Gold ETF), and IWM (Russell 2000 ETF). Historically, closing prices, returns, and associated metrics were retrieved for portfolio performance evaluation under uncertainty. Data extraction and pre-processing were conducted using the Python library *yfinance* (<https://finance.yahoo.com/>). Supplementary analysis and statistical computations were performed using standard libraries, including *pandas*, *numpy*, and *matplotlib*.

Conflict of Interest

The author declares no conflict of interest related to this study.

Author Contribution Statement

The author solely conducted all aspects of this research, including conceptualization, methodology, data analysis, manuscript writing, and final approval of the version to be published.

Disclosure Statement

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Appendix A: Raw Decision Matrix

The initial decision matrix contains the financial indicators used to evaluate the ETF alternatives.

Criteria classification

- a. Benefit criteria
 - CAGR

- Sharpe Ratio
 - Liquidity
- b. Cost criteria
- Volatility
 - Maximum Drawdown

Table A1: ETF Alternatives

Ticker	CAGR	Volatility	Sharpe	MaxDrawdown	Liquidity
SPY	0.160464	0.175139	0.942077	-0.244964	7.54E+07
QQQ	0.175586	0.23353	0.813342	-0.351187	4.88E+07
GLD	0.106739	0.154086	0.739091	-0.220022	8.32E+06
IWM	0.098396	0.231151	0.523748	-0.319116	3.03E+07
TLT	-0.101583	0.162136	-0.583224	-0.483511	2.67E+07

Source: Author analysis

Appendix B: Normalization Procedure

To ensure comparability across criteria, the decision matrix is normalized.

Benefit Criteria

$$N_{ij} = \frac{X_{ij}}{\max(X_j)} \quad (\text{B1})$$

Cost Criteria

$$N_{ij} = \frac{\min(X_j)}{X_{ij}} \quad (\text{B2})$$

where,

X_{ij} = value of alternative i under criterion j

N_{ij} = normalized value

Appendix C: Fuzzy Membership Transformation

Normalized values are converted into linguistic categories using fuzzy logic proposed by Lotfi A. Zadeh.

Table C1: Fuzzy Score

Normalized Value	Linguistic Variable	Fuzzy Score
0.0–0.2	Very Low	1
0.2–0.4	Low	2
0.4–0.6	Medium	3
0.6–0.8	High	4
0.8–1.0	Very High	5

Source: Author Analysis

These fuzzy values capture uncertainty and qualitative interpretation of financial performance indicators.

Appendix D: Criterion Ranking Matrix

After fuzzy transformation, ETFs are ranked under each criterion.

Table D1: Criterion Ranking Matrix

Ticker	CAGR Rank	Sharpe Rank	Liquidity Rank	Volatility Rank	Drawdown Rank
SPY	2	1	1	2	2
QQQ	1	2	2	4	4
GLD	3	3	5	1	1
IWM	4	4	3	5	3
TLT	5	5	4	3	5

Source: Author analysis

Appendix E: Weighted Rank Aggregation

The final ranking is obtained using the Structured Ranking Process (SRP) aggregation method based on weighted ranks introduced by Ralph W. Edwards.

$$WR_i = \sum_{j=1}^n w_j R_{ij} \tag{E1}$$

where:

WR_i = weighted rank score of alternative i

w_j = weight of criterion j

R_{ij} = rank of alternative i under criterion j

Appendix F: Final Ranking Outcome

The results demonstrate that SPY achieves the highest overall ranking, reflecting superior risk-adjusted performance and liquidity characteristics.

Table F1: Final Ranking Outcome

Ticker	Weighted Rank Sum	Final Rank
SPY	32	1
QQQ	35	2
TLT	48	3
IWM	51	4
GLD	59	5

Source: Author analysis